

# Package ‘resde’

May 19, 2023

**Title** Estimation in Reducible Stochastic Differential Equations

**Version** 1.1

**Description** Maximum likelihood estimation for univariate reducible stochastic differential equation models. Discrete, possibly noisy observations, not necessarily evenly spaced in time. Can fit multiple individuals/units with global and local parameters, by fixed-effects or mixed-effects methods. Ref.: Garcia, O. (2019) “Estimating reducible stochastic differential equations by conversion to a least-squares problem”, Computational Statistics 34(1): 23-46, <doi:10.1007/s00180-018-0837-4>.

**License** GPL (>= 2)

**Encoding** UTF-8

**RoxygenNote** 7.2.3

**Imports** stats, Deriv, nlme, methods

**Suggests** knitr

**VignetteBuilder** knitr

**URL** <https://github.com/ogarciav/resde/>

**BugReports** <https://github.com/ogarciav/resde/issues>

**NeedsCompilation** no

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**Repository** CRAN

**Date/Publication** 2023-05-19 17:20:09 UTC

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resde-package	<b>resde</b> - <i>Parameter estimation in reducible SDE models.</i>
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## Description

The main functions for model fitting are `sdemodel()` and `sdefit()`. First, specify the model structure in `sdemodel()`, including the variable transformation, any re-parameterizations, initial condition, and the presence or not of process, measurement, and initial condition noise. Then, fit the model with `sdefit()`, indicating the data to be used and starting parameter values for the iterations. For hierarchical models, one must also indicate which are the global and local parameters, and if fixed locals or a mixed effects method should be used.

Some auxilliary functions include the Box-Cox transformation `bc()`, and the *unified transformation* `unitran()`.

For detailed usage see the vignette: `vignette("resde-vignette", package="resde")`.

## Author(s)

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## References

Garcia, O. (2019) "Estimating reducible stochastic differential equations by conversion to a least-squares problem". *Computational Statistics* 34(1), 23-46. doi:[10.1007/s0018001808374](https://doi.org/10.1007/s0018001808374)

## See Also

Useful links:

- <https://github.com/ogarciav/resde/>
- Report bugs at <https://github.com/ogarciav/resde/issues>

## Examples

```
# Richards model dH^c = b(a^c - H^c) dt + s dW for tree heights
tree1 <- subset(Loblolly, Seed == Seed[1]) # first tree
m <- sdemodel(~x^c, beta0=~b*a^c, beta1=~-b, mum=0) # no measurement error
sdefit(m, x="height", t="age", data=tree1, start=c(a=70, b=0.1, c=0.5))
```

---

bc	<i>Box-Cox transformation</i>
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### Description

These functions calculate the Box-Cox transformation, its inverse, and derivative.

### Usage

```
bc(x, lambda)
```

```
bc_inv(y, lambda)
```

```
bc_prime(y, lambda)
```

### Arguments

x, y            Numeric vector (x must be  $\geq 0$ ).

lambda         Numeric scalar, power parameter.

### Details

bc() uses `expm1()`, which is more accurate for small lambda than a more "obvious" alternative like

```
if (abs(lambda) < 6e-9) log(y)
else (y^lambda - 1) / lambda
```

The difference might be important in optimization applications. See example below. Similarly, `bc_inv()` uses `log1p()`.

### Value

bc(): Returns the transform value(s).

bc\_inv(): Computes the inverse of bc().

bc\_prime(): Gives the derivative of bc() with respect to y.

### Functions

- `bc()`: The Box-Cox transformation
- `bc_inv()`: Inverse of the Box-Cox transformation
- `bc_prime()`: Derivative of the Box-Cox transformation

**Examples**

```
bc(0.5, 1.5)
bc(1, 0)
obvious <- function(lambda){(0.6^lambda - 1) / lambda} # at y = 0.6
plot(obvious, xlab="lambda", xlim=c(1e-6, 1e-9), log="x")

bc_inv(-0.4, 1.5)
bc_inv(0, 0)

bc_prime(0.5, 1.5)
bc_prime(1, 0)
```

---

sdefit

*Fit SDE model*


---

**Description**

ML estimation of parameters for a reducible SDE

**Usage**

```
sdefit(model, x, t, unit=NULL, data=NULL, start=NULL,
       global=NULL, local=NULL, known=NULL, method="nls",
       control=NULL, phi=NULL, phiprime=NULL)
```

**Arguments**

model	Model specification, as produced by <code>sdemodel()</code> .
x, t	Vectors with variables, or names of columns in data frame.
unit	If applicable, unit id vector, or name of its column in data frame.
data	Data frame, if data not given directly in x, t, unit.
start	Named vector or named list with starting parameter values for non-hierarchical models. They can also be given in global.
global	Named vector or list of global parameters and their starting values for hierarchical models. Can also contain starting values for non-hierarchical models.
local	Named vector or list of local parameters and their starting values for hierarchical models. The value can be a vector with values for each unit, or a single scalar that applies to all the units.
known	Named vector or list with any parameters that should be fixed at given values.
method	'nls' for non-hierarchical models (default). For hierarchical models it can be 'nls', for fixed locals, or 'nlme' for mixed effects.
control	Optional control list for <code>nls()</code> or <code>nlme()</code> .
phi	Optional transformation function. If NULL (default), it is automatically generated.
phiprime	Optional derivative function. If NULL (default), it is automatically generated.

**Value**

List with two components: a list `fit` containing the output from the optimizer (nls or nlme), and a list `more` containing sigma estimates, log-likelihood, AIC and BIC. Note that in `fit`, "residual sum-of-squares" corresponds to `uvector`, not to `x` or `y`. Same for nls and nlme methods like `fitted` or `residuals` applied to `fit`.

**Examples**

```
m <- sdemodel(phi=~x^c, beta0=~b*a^c, beta1=~-b)
mod1 <- sdefit(m, "height", "age", data=Loblolly[Loblolly$Seed=="301",],
              start=c(a=70, b=0.1, c=1))
mod2 <- sdefit(m, "height", "age", "Seed", Loblolly, global=c(b=0.1, c=0.5),
              local=c(a=72))
```

---

sdemodel

*Model specification*


---

**Description**

Specify transformation and re-parametrizations for reducible SDE model.

**Usage**

```
sdemodel(phi=~x, phiprime=NULL, beta0=~beta0, beta1=~beta1,
         t0=0, x0=0, mu0=0, mup=1, mum=1)
```

**Arguments**

<code>phi</code>	Transformation formula $y = \varphi(x, parameters)$ .
<code>phiprime</code>	Optional formula for derivative of <code>phi</code> .
<code>beta0, beta1</code>	Optional formulas or constants, possibly giving a re-parameterization,.
<code>t0, x0</code>	Formulas or constants for the initial condition.
<code>mu0</code>	Formula or constant for the initial condition $\sigma_0$ multiplier.
<code>mup, mum</code>	Formulas or constants for the process and measurement $\sigma$ multipliers.

**Value**

List with model specification, to be used by `sdefit()`.

**Examples**

```
richards <- sdemodel(phi=~x^c, beta0=~b*a^c, beta1=~-b, mum=0)
```

---

sdemodel\_display      *Display the model specification*

---

**Description**

Display the model specification

**Usage**

```
sdemodel_display(model)
```

**Arguments**

model                  SDE model specification, as produced by sdemodel()

**Value**

Invisibly returns its argument

**Examples**

```
mod <- sdemodel(); sdemodel_display(mod)
```

---

str2fun\_theta          *String to function, with parameters in theta*

---

**Description**

Normally not called by the user directly, used by `sdefit()`. Converts an expression, in a character string, to a function.

**Usage**

```
str2fun_theta(s)
```

**Arguments**

s                      String representation of a function of x and parameters

**Value**

Function of x and theta, theta being a named vector or list of parameters.

**Examples**

```
str2fun_theta("x^c / a")
```

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unitran	<i>Unified transformation</i>
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---

### Description

Calculates a variable transformation that produces various growth curve models, depending on the values of two shape parameters, alpha and beta. Models can also be specified by name. Uses `bc()`, `bc_inv()`, `bc_prime()`.

### Usage

```
unitran(x, name=NULL, par=NULL, alpha=NULL, beta=NULL, reverse="auto")
```

```
unitran_inv(y, name=NULL, par=NULL, alpha=NULL, beta=NULL, reverse="auto")
```

```
unitran_prime(x, name=NULL, par=NULL, alpha=NULL, beta=NULL, reverse="auto")
```

### Arguments

<code>x, y</code>	Variable to be transformed, x must be between 0 and 1.
<code>name</code>	Optional model name, case-insensitive, in quotes. One of Richards, monomolecular, Mitscherlich, Bertalanffy, Gompertz, logistic, Levacovic, Weibull, Korf, exponential, Schumacher, Hosfeld.
<code>par</code>	Model parameter, if needed and model name supplied.
<code>alpha, beta</code>	Shape parameters, if the model is not specified by name.
<code>reverse</code>	Reverse x and t axes? One of "yes", "no", "auto". With "auto", axes are reversed as necessary for an upper asymptote. (i.e., if $\alpha \leq 0$ and $\beta > 0$ ).

### Value

`unitran()`: Transformed x, i.e.,  $y = \varphi(x)$ .

`unitran_inv()`: Inverse of `unitran()`,  $x = \varphi^{-1}(y)$ .

`unitran_prime()`: Derivative of `unitran()`,  $y' = \varphi'(x)$ .

### Functions

- `unitran()`: Unified transformation.
- `unitran_inv()`: Inverse of `unitran()`.
- `unitran_prime()`: Derivative of `unitran()` with respect to x.

### Examples

```
curve(unitran(x, "Gompertz")) # same as unitran(x, alpha=0, beta=0)
curve(unitran_inv(y, "logistic"), xname="y", from=-4, to=4)
curve(unitran_prime(x, "logistic"))
```

---

userphi *Examples of optional external transformation and derivative functions*

---

### Description

Templates for user-supplied transformation and derivative functions, used by `sdefit()` if specified in parameters `phi` and/or `phiprime`. To be completed by the user.

### Usage

```
userphi(x, theta)
```

```
userphiprime(x, theta)
```

### Arguments

<code>x</code>	Numeric vector, variable to be transformed.
<code>theta</code>	Named list of transformation parameters

### Value

Transformed variable  
Transformation derivative

### Functions

- `userphi()`: transformation
- `userphiprime()`: derivative

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uvector *ML estimation vector for reducible SDEs*

---

### Description

These functions are not normally called directly by the user. Function `uvector()` is used by `sdefit()`. Function `uvector_noh()` is a more limited version, maintained for documentation purposes. Function `logdet_and_v()` is used by `uvector()` and `uvector_noh()`.

### Usage

```
uvector(x, t, unit = NULL, beta0, beta1, eta, eta0, x0, t0, lambda,
        mum = 1, mu0 = 1, mup = 1, sorted = FALSE, final = FALSE)
```

```
uvector_noh(x, t, beta0, beta1, eta, eta0, x0, t0, lambda, final = FALSE)
```

```
logdet.and.v(cdiag, csub = NULL, z)
```



**Arguments**

<code>x, t</code>	Data vectors
<code>unit</code>	Unit id vector, if any.
<code>beta0, beta1, eta, eta0, x0, t0</code>	SDE parameters or re-parameterizations.
<code>lambda</code>	Named list of parameters(s) for <code>phi()</code> , possibly local vectors.
<code>mum, mu0, mup</code>	Optional $\sigma$ multipliers.
<code>sorted</code>	Data already ordered by increasing <code>t</code> ?
<code>final</code>	Mode, see below.
<code>cdiag</code>	Vector with the diagonal elements $c_{ii}$ of $C$ .
<code>csub</code>	Vector with sub-diagonal $c_{i,i-1}$ for $i > 1$ .
<code>z</code>	A numeric vector

**Details**

`uvector()` and `uvector_noh()` calculate a vector of residuals for sum of squares minimization by `nls()` or `nlme()`. The first one works both for single-unit and for bilevel hierarchical models. It is backward-compatible with `uvector_noh()`, which is only for single-unit models but simpler and easier to understand. They require a transformation function `phi(x, theta)`, and a function `phiprime(x, theta)` for the derivative  $dy/dx$ , where `theta` is a list containing the transformation parameters.

`logdet_and_v()` calculates  $\log[\det(L)]$  and  $v = L^{-1}z$ , where  $C = LL'$ , with  $L$  lower-triangular.

The three functions are essentially unchanged from García (2019) <[doi:10.1007/s001800180837-4](https://doi.org/10.1007/s001800180837-4)>, except for a somewhat safer computation for very small `beta1`, and adding in `logdet_and_v()` a shortcut for when  $L$  is diagonal (e.g., when  $\sigma_m = 0$ ). The transformation functions `phi` and `phiprime` can be passed as globals, as in the original, or in an environment named `trfun`.

**Value**

`uvector()` and `uvector_noh()`: If `final = FALSE` (default), return a vector whose sum of squares should be minimized over the parameters to obtain maximum-likelihood estimates. If `final = TRUE`, passing the ML parameter estimates returns a list with the sigma estimates, the maximized log-likelihood, and AIC and BIC criteria..

`logdet_and_v()`: List with elements `logdet` and `v`.

**Functions**

- `uvector()`: Estimation vector, general
- `uvector_noh()`: Estimation vector, non-hierarchical
- `logdet.and.v()`: Logarithm of determinant, and  $v$  vector

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