

Package ‘gallery’

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Type Package

Title Generate Test Matrices for Numerical Experiments

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Maintainer Thomas Hsiao <thomas.hsiao@emory.edu>

Description Generates a variety of structured test matrices commonly used in numerical linear algebra and computational experiments. Includes well-known matrices for benchmarking and testing the performance, stability, and accuracy of linear algebra algorithms. Inspired by 'MATLAB' 'gallery' functions.

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Imports Matrix, pracma (>= 2.2.5)

Suggests testthat (>= 2.1.0), knitr, rmarkdown

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Author Thomas Hsiao [aut, cre] (<<https://orcid.org/0009-0005-7848-7992>>)

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binomial_matrix	<i>Create binomial matrix</i>
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Description

Binomial matrix: an N-by-N multiple of an involutory matrix with integer entries such that $A^2 = 2^{N-1} I_N$. Thus $B = A \cdot 2^{-(N-1)/2}$ is involutory, that is $B^2 = EYE(N)$

Usage

binomial_matrix(n)

Arguments

n row dimension

Value

a binomial matrix, which is a multiple of involutory matrix

cauchy_matrix	<i>Create Cauchy matrix</i>
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Description

Arguments x and y are vectors of length n. $C[i, j] = 1 / (x[i] + y[j])$

Usage

cauchy_matrix(x, y = NULL)

Arguments

x vector of length n
 y vector of length n

Value

a Cauchy matrix

chebspec	<i>Create Chebyshev spectral differentiation matrix</i>
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Description

Chebyshev spectral differentiation matrix of order n. k determines the character of the output matrix. For either form, the eigenvector matrix is ill-conditioned.

Usage

chebspec(n, k = NULL)

Arguments

n order of the matrix.
 k k=0 is the default, no boundary conditions. The matrix is similar to a Jordan block of size n with eigenvalue 0. If k=1, the matrix is nonsingular and well-conditioned, and its eigenvalues have negative real parts.

Value

Chebyshev spectral differentiation matrix

chebvand	<i>Creating Vandermonde-like matrix for the Chebyshev polynomials</i>
----------	---

Description

Produces the (primal) Chebyshev Vandermonde matrix based on the points p. $C[i, j] = T_{i-1}(p[j])$, where T_{i-1} is the Chebyshev polynomial of degree $i-1$

Usage

chebvand(p, m = NULL)

Arguments

p points to evaluate. If a scalar, then p equally spaced points on $[0, 1]$ are used.
 m number of rows of the matrix. `chebvand(p, m)` is the rectangular version of `chebvand(p)` with m rows.

Value

Vandermonde-like matrix for the Chebyshev polynomials

chow *Creating singular Toeplitz lower Hessenberg matrix*

Description

returns matrix $A = H(\alpha) + \delta * \text{EYE}$, such that $H[i, j] = \alpha^{(i-j+1)}$.

Usage

`chow(n, alpha = 1, delta = 0)`

Arguments

n order of the matrix
 alpha defaults to 1
 delta defaults to 0

Value

Singular Toeplitz lower Hessenberg matrix

circul *Create circulant matrix*

Description

Each row is obtained from the previous by cyclically permuting the entries one step forward. A special Toeplitz matrix in which diagonals "wrap around"

Usage

`circul(v)`

Arguments

v first row of the matrix. If v is a scalar, then $C = \text{circul}(1:v)$

Value

a circulant matrix whose first row is the vector v

clement

Create Clement tridiagonal matrix with zero diagonal entries

Description

Returns an n -by- n tridiagonal matrix with zeros on the main diagonal. For $k=0$, A is nonsymmetric. For $k=1$, A is symmetric

Usage

`clement(n, k = 0)`

Arguments

n	order of matrix
k	0 indicates symmetric matrix, 1 asymmetric

Value

Clement tridiagonal matrix with zero diagonal entries

compar

Create comparison matrix A

Description

For $k=0$, if $i=j$, $A[i,j]=\text{abs}(B[i,j])$ and $A[i,j]=-\text{abs}(B[i,j])$ otherwise. For $k=1$, A replaces each diagonal element of B with its absolute value, and replaces each off-diagonal with the negative of the largest absolute value off-diagonal in the same row.

Usage

`compar(B, k = 0)`

Arguments

B	input matrix
k	decides what matrix to return

Value

Comparison matrix

cycol *Create matrix A whose columns repeat cyclically*

Description

Returns an n-by-n matrix with cyclically repeating columns where one cycle consists of the columns defined by `randn(n,k)`. Thus, the rank of matrix A cannot exceed k, and k must be scalar.

Usage

```
cycol(n, k, m = NULL)
```

Arguments

n	number of columns of matrix
k	upper limit of rank
m	number of rows of matrix

Value

Matrix whose columns repeat cyclically

dorr *Create Dorr matrix*

Description

Returns a n-by-n row diagonally dominant, tridiagonal matrix that is ill-conditioned for small non-negative values of theta. The default value of theta is 0.01.

Usage

```
dorr(n, theta = 0.01)
```

Arguments

n	order of matrix
theta	determines conditionality. Ill-conditioned when theta is nonnegative.

Value

Dorr matrix of class 'dgcMatrix'.

dramadah	<i>Create anti-Hadamard matrix A</i>
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Description

Returns a n-by-n nonsingular matrix of 0's and 1's. With large determinant or inverse. If k=1, A is Toeplitz and $\text{abs}(\det(A))=1$. If k=2, A is upper triangular and Toeplitz. If k=3, A has maximal determinant among (0,1) lower Hessenberg matrices. Also is Toeplitz.

Also known as an anti-Hadamard matrix.

Usage

```
dramadah(n, k = 1)
```

Arguments

n	order of matrix
k	decides type of matrix returned.

Value

Matrix of zeros and ones

fiedler	<i>Create Fiedler matrix</i>
---------	------------------------------

Description

Fiedler matrix that has a dominant positive eigenvalue and all others are negative

Usage

```
fiedler(c)
```

Arguments

c	N-vector. If c is a scalar, then returns fiedler(1:c)
---	---

Value

a symmetric dense matrix A with a dominant positive eigenvalue and all others are negative.

 forsythe

Create Forsythe matrix or perturbed Jordan block

Description

Returns a n-by-n matrix equal to the Jordan block with eigenvalue lambda, except that $A[n, 1] = \alpha$.

Usage

```
forsythe(n, alpha = .Machine$double.eps, lambda = 0)
```

Arguments

n	order of matrix
alpha	value of perturbation at $A[n, 1]$
lambda	eigenvalue of Jordan block

Value

Forsythe matrix or perturbed Jordan block

frank

Frank matrix of order N

Description

Frank matrix of order N. It is upper Hessenberg with determinant 1.

Usage

```
frank(n, k = 0)
```

Arguments

n	order of the matrix
k	If k is 1, the elements are reflected about the anti-diagonal.

Value

Frank matrix with ill-conditioned eigenvalues.

grcar	<i>Create Toeplitz matrix with sensitive eigenvalues</i>
-------	--

Description

Eigenvalues are sensitive.

Usage

```
grcar(n, k = NULL)
```

Arguments

n	dimension of the square matrix
k	number of superdiagonals of ones

Value

n-by-n Toeplitz matrix with -1 on subdiagonal, 1 on diagonal, and k superdiagonals of 1s.

hanowa	<i>Hanowa matrix</i>
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Description

Matrix whose eigenvalues lie on vertical plane in complex plane. Returns a 2-by-2 block matrix with four n/2 by n/2 blocks. n must be an even integer.

```
[d*eye(m) -diag(1:m), diag(1:m) d*eye(m)]
```

Usage

```
hanowa(n, d = NULL)
```

Arguments

n	order of matrix
d	value of main diagonal

Value

Matrix whose eigenvalues lie on a vertical line in the complex plane.

invol *Involutory matrix (a matrix that is its own inverse)*

Description

a n-by-n involutory matrix and ill-conditioned. It is a diagonally scaled version of a Hilbert matrix.

Usage

```
invol(n)
```

Arguments

n order of matrix

Value

Involutory matrix (a matrix that is its own inverse).

jordbloc *Create Jordan block matrix*

Description

Returns a n-by-n Jordan block with eigenvalue lambda. The default is 1.

Usage

```
jordbloc(n, lambda = 1)
```

Arguments

n order of matrix
lambda eigenvalue of Jordan block

Value

Jordan block matrix

lauchli	<i>Create Lauchli Matrix</i>
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Description

the $(N + 1) \times (N)$ matrix `[ones(1,n); mu*eye(n)]`. Well-known example in least squares of the danger of forming $t(A)$

Usage

```
lauchli(n, mu = NULL, sparse = FALSE)
```

Arguments

n	number of columns
mu	constant applied to identity
sparse	whether matrix should be sparse

Value

Lauchli rectangular matrix.

lehmer	<i>Create Lehmer matrix</i>
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Description

the symmetric positive-definite matrix such that $A[i,j] = i/j$, for $j \geq i$

Usage

```
lehmer(n)
```

Arguments

n	order of matrix
---	-----------------

Value

Lehmer symmetric positive definite matrix.

leslie	<i>Create Leslie population model matrix</i>
--------	--

Description

N by N matrix from Leslie population model with average birth and survival rates.

Usage

```
leslie(a, b = NULL, sparse = FALSE)
```

Arguments

a	average birth numbers (first row)
b	survival rates (subdiagonal)
sparse	whether to return a sparse matrix

Value

N by N Leslie population model matrix

minij	<i>Symmetric positive definite matrix MIN(i, j)</i>
-------	---

Description

The N-by-N SPD matrix with $A[i, j] = \min(i, j)$

Usage

```
minij(n)
```

Arguments

n	order of the matrix
---	---------------------

Value

Symmetric positive definite matrix with entries $A[i, j] = \min(i, j)$

spdiags *Create sparse diagonal matrix*

Description

Creates a sparse representation of multiple diagonal matrix

Usage

```
spdiags(A, d, m, n)
```

Arguments

A	matrix where columns correspond to the desired diagonals
d	indices of the diagonals to be filled in. 0 is main diagonal. -1 is first subdiagonal and +1 is first superdiagonal.
m	row dim
n	col dim

Value

sparse diagonal matrix of class 'dgcMatrix'

tridiag *Create sparse tridiagonal matrix*

Description

Create a sparse tridiagonal matrix of dgcMatrix class.

Usage

```
tridiag(n, x = NULL, y = NULL, z = NULL)
```

Arguments

n	dimension of the square matrix
x	subdiagonal (-1)
y	diagonal (0)
z	superdiagonal (+1)

Value

Sparse tridiagonal matrix of class 'dgcMatrix'

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